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PROBLEM OF SELECTING ZERO APPROXIMATION FOR POSITION OF AN ORIENTED SATELLITE USING NON-DIPOLE APPROXIMATION OF THE GEOMAGNETIC FIELD

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16. Abstract

A program of statistical smoothing to the tenth power exclusively was developed; then statistical smoothing for two oriented satellites in circular orbits at altitudes of 250 to 600 cm was carried out. This study proceeds on the basis of the author's other studies cited in the same monograph.

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THE PROBLEM OF SELECTING A ZERO APPROXIMATION FOR THE ANGULAR POSITION OF AN ORIENTED SATELLITE USING NON-DIPOLE APPROXIMATION OF THE GEOMAGNETIC FIELD

V. S. Novoselov

1. The Required Number of Terms of Expansion in the Trigonometric Approximation of Projections of Geomagnetic Field Intensity /139

The investigation conducted in study [1] showed that by using a trigonometric approximation we can derive rather simple formulas for the zero approximation of orientation angles. If we retain terms of the second power in the approximating formulas for the angles, then terms up to the third power exclusively in H_x, H_y and H_z will have rather large coefficients. The research stated in study [2] shows that projections of intensity have in the approximating formulas terms of power one greater than the greatest power of the polynomial of trigonometric approximation of the angles of AES position.

The question arises as to what order the coefficients of trigonometric approximation of intensity projections of the geomagnetic field have under actual AES flight conditions. An explanation of this question was given by the following numerical experiment. A program of statistical smoothing to the tenth power exclusively was developed. Then statistical smoothing for two oriented satellites in circular orbits (virtually circular) at altitudes of 250-600 cm was carried out.

Analysis of the results obtained shows that the most substantial coefficients on the order of 0.01 have only terms up to the third power. This affirms that the suggestion of smoothing of orientation angles in terms of trigonometric polynomials by powers of the argument of latitude to the second power exclusively for low satellites is totally acceptable. This analysis showed further that the largest coefficients on the order of 0.1 have terms corresponding to the dipole approximation of the geomagnetic field. But non-dipole coefficients H^0_{ik} of the model of the field have the same order as the corresponding coefficients H_{ik} for readings of magnetometric sensors. Since the methods of selection of the zero approximation substantially use all H_{ik}, then to raise the accuracy and reliability of the definition of the zero approximation we must use transformations in study [1] to substitute an /140 approximating model (as follows) for the dipole model of the field:

$$\left.\begin{aligned}
H_{x_0}&=H^0_{11}+\sum_{n=1}^{3}(H^0_{1,2n}\sin nu+H^0_{1,2n+1}\cos nu),\\
H_{y_0}&=H^0_{21}+\sum_{n+1}^{3}(H^0_{2,2n}\sin nu+H^0_{2,2n+1}\cos nu),\\
H_{z_0}&=H^0_{31}+\sum_{n=1}^{3}(H^0_{3,2n}\sin nu+H^0_{3,2n+1}\cos nu).
\end{aligned}\right| \tag{1}$$

2. Algorithm of Solution of the Problem to Within Three Additionally Assigned Coefficients

Given for the selected orbit of an actual AES that we have conducted statistical smoothing of the model of the geomagnetic field in terms of formulas (1). Given further that we conduct statistical processing of magnetometer readings by formulas of the type (5)-(7) of study [2] with retention of the basic terms

$$\left.\begin{aligned}
H_x&=H_{11}+\sum_{n=1}^{3}(H_{1,2n}\sin nu+H_{1,2n+1}\cos nu),\\
H_y&=H_{21}+\sum_{n=1}^{3}(H_{2,2n}\sin nu+H_{2,2n+1}\cos nu),\\
H_z&=H_{31}+\sum_{n=1}^{3}(H_{3,2n}\sin nu+H_{3,2n+1}\cos nu).
\end{aligned}\right| \tag{2}$$

For aircraft angles of orientation, the formulas of transfer from projection of geomagnetic intensity onto orbital axes to projections of intensity onto the axes of constructive axes will be written thus:

$$
\begin{aligned}
H_x &= H_{x_0}\cos\psi\cos\theta + H_{y_0}\sin\psi\cos\theta - H_{z_0}\sin\theta,\\
H_y &= H_{x_0}(\cos\psi\sin\theta\sin\varphi - \sin\psi\cos\varphi) + H_{y_0}(\sin\psi\sin\theta\sin\varphi +\\
&\quad + \cos\psi\cos\varphi) + H_{z_0}\cos\theta\sin\varphi,\\
H_z &= H_{x_0}(\cos\psi\sin\theta\cos\varphi + \sin\psi\sin\varphi) + H_{y_0}(\sin\psi\sin\theta\cos\varphi -\\
&\quad - \cos\psi\sin\varphi) + H_{z_0}\cos\theta\cos\varphi.
\end{aligned}
\qquad (3)
$$

In order to explain the theoretical aspect in using non-linear equations (3), we will limit ourselves in these equations to quadratic terms relative to values of orientation angles

$$
\begin{aligned}
H_x &= H_{x_0}\left(1 - \frac{\psi^2}{2} - \frac{\theta^2}{2}\right) + H_{y_0}\psi - H_{z_0}\theta,\\
H_y &= H_{x_0}(-\psi + \theta\varphi) + H_{y_0}\left(1 - \frac{\varphi^2}{2} - \frac{\psi^2}{2}\right) + H_{z_0}\varphi,\\
H_z &= H_{x_0}(\theta + \psi\varphi) + H_{y_0}(-\varphi + \psi\theta) + H_{z_0}\left(1 - \frac{\theta^2}{2} - \frac{\varphi^2}{2}\right).
\end{aligned}
\qquad (4)
$$

Let us assume the following trigonometric approximation for /141 the angles:

$$
\begin{aligned}
\psi &= D_{11} + \sum_{m=1}^{2}(D_{1,2m}\sin mu + D_{1,2m+1}\cos mu),\\
\varphi &= D_{21} + \sum_{m=1}^{2}(D_{2,2m}\sin mu + D_{2,2m+1}\cos mu),\\
\theta &= D_{31} + \sum_{m=1}^{2}(D_{3,2m}\sin mu + D_{3,2m+1}\cos mu).
\end{aligned}
\qquad (5)
$$

If expressions (5) are substituted in relation (4), then coefficients in H_{x0}, H_{y0}, H_{z0} on the right sides of relations (4) will be trigonometric polynomials, containing trigonometric functions of the argument of latitude to the fourth power. Here we must use the following elementary formulas of trigonometry:

$$
\begin{aligned}
\sin nu \sin mu &= \frac{1}{2}\cos(n-m)u - \frac{1}{2}\cos(n+m)u, \\
\cos nu \sin mu &= \frac{1}{2}\sin(n+m)u - \frac{1}{2}\sin(n-m)u, \\
\sin nu \cos mu &= \frac{1}{2}\sin(n+m)u + \frac{1}{2}\sin(n-m)u, \\
\cos nu \cos mu &= \frac{1}{2}\cos(n-m)u + \frac{1}{2}\cos(n+m)u.
\end{aligned}
\tag{6}
$$

Let us then substitute in the transformed equations (4) the notions of (1) and (2). On the left of the new equations we will have the trigonometric polynomials to the third power; on the right--to the seventh.

Le us equate the left and right in the obtained identities for the coefficients with identical trigonometric functions. We will derive 45 equations which contain 15 unknown coefficients of trigonometric approximation of angles D_{p1}, $D_{p,2m}$, $D_{p,\ 2m+1}$ ($p = 1, 2, 3$) both linearly and in the form of a quadratic representation (the product of two coefficients and squares of the coefficients).

In the phase of oriented motion, the orientation angles are on the order of 5-10°, i.e., on the order of 0.1. Thus the main terms will be linear. The linear parts of these equations are derived if we ignore the quadratic terms in formulas (4):

$$
\left.\begin{aligned}
H_x &= H_{x_0} + H_{y_0}\psi - H_{z_0}\theta;\\
H_y &= -H_{x_0}\psi + H_{y_0} + H_{z_0}\varphi,\\
H_z &= H_{x_0}\theta - H_{y_0}\varphi + H_{z_0}.
\end{aligned}\right| \tag{7}
$$

Let us substitute in (7) expressions (1) and (5) and use formulas (6). We cite for example the first of the derived rela- /142 tionships:

$$
\begin{aligned}
H_x = {} & H^0_{11} + D_{11}H^0_{21} - D_{31}H^0_{31} + \sum_{m=1}^{2} \sin mu\,(H^0_{1,2m} + D_{11}H^0_{2,2m} -\\
& - D_{31}H^0_{3,2m} + D_{1,2m}H^0_{21} - D_{3,2m}H^0_{31}) + \sum_{m=1}^{2} \cos mu\,(H^0_{1,2m+1} +\\
& + D_{11}H^0_{2,2m+1} - D_{31}H^0_{3,2m+1} + D_{1,2m+1}H^0_{21} - D_{3,2m+1}H^0_{31}) +\\
& + \sin 3u\,(H^0_{1,6} + D_{11}H^0_{2,6} - D_{31}H^0_{3,6}) + \cos 3u\,(H^0_{1,7} + D_{11}H^0_{2,7} -\\
& - D_{31}H^0_{3,7}) + \frac{1}{2}\sum_{n=1}^{3}\sum_{m=1}^{2} \sin (n-m)\,u\,(D_{1,2m+1}H^0_{2,2n} - D_{3,2m+1}H^0_{3,2n} -\\
& - D_{1,2m}H^0_{2,2n+1} + D_{3,2m}H_{3,2n+1}) + \frac{1}{2}\sum_{n=1}^{3}\sum_{m=1}^{2} \cos (n-m)\,u \times\\
& \times (D_{1,2m}H^0_{2,2n} - D_{3,2m}H^0_{3,2n} + D_{1,2m+1}H^0_{2,2n+1} - D_{3,2m+1}H^0_{3,2n+1}) +\\
& + \frac{1}{2}\sum_{n=1}^{3}\sum_{m=1}^{2} \sin (n+m)\,u\,(D_{1,2m}H^0_{2,2n+1} - D_{3,2m}H^0_{3,2n+1} +\\
& + D_{1,2m+1}H^0_{2,2n} - D_{3,2m+1}H^0_{3,2n}) + \frac{1}{2}\sum_{n=1}^{3}\sum_{m=1}^{2} \cos (n+m)\,u \times\\
& \times (-D_{1,2m}H^0_{2,2n} + D_{3,2m}H^0_{3,2n} + D_{1,2m+1}H^0_{2,2n+1} - D_{3,2m+1}H^0_{3,2n+1}).
\end{aligned} \tag{8}
$$

As a result of equating terms with like trigonometric functions, we will have 33 linear equations to define 15 unknown coefficients D_{p1}, $P_{p,2m}$, $D_{p,2m+1}$.

These equations will be rather unwieldy. For example, let us write the first two equations:

$$
\begin{aligned}
H_{11} = {} & H^0_{11} + D_{11}H^0_{21} + \frac{1}{2}D_{12}H^0_{32} + \frac{1}{2}D_{13}H^0_{23} + \frac{1}{2}D_{14}H^0_{24} +\\
& + \frac{1}{2}D_{15}H^0_{25} - D_{31}H^0_{31} - \frac{1}{2}D_{32}H^0_{32} - \frac{1}{2}D_{33}H^0_{33} -\\
& - \frac{1}{2}D_{34}H^0_{34} - \frac{1}{2}D_{35}H^0_{35};
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
H_{12}=&H^0_{12}+D_{11}H^0_{22}+D_{12}\left(H^0_{21}-\frac{1}{2}H^0_{25}\right)+\frac{1}{2}D_{13}H^0_{24}+\\
&+\frac{1}{2}D_{14}(H^0_{23}-H^0_{27})+\frac{1}{2}D_{15}(H^0_{26}-H^0_{22})-\\
&-D_{31}H^0_{32}-D_{32}\left(H^0_{31}-\frac{1}{2}H^0_{35}\right)-\frac{1}{2}D_{33}H^0_{34}-\\
&-\frac{1}{2}D_{34}(H^0_{33}+H^0_{37})-\frac{1}{2}D_{35}(-H^0_{32}+H^0_{36}).
\end{aligned}
\qquad (10)
$$

But not all values of $H^0_{p,1}$, $H^0_{p,2n}$, $H^0_{p,2n+1}$ on the right sides of the derived equations will be of one order. As we showed in section 1, the coefficients of a dipole approximation $H^0_{1,3}$, $H^0_{2,1}$, /143 $H^0_{3,2}$ have an order of 0.1, while the other coefficients have an order of 0.01. The unknown coefficients of the trigonometric approximation also have an order of 0.01. Since the quadratic terms relative to these unknowns have already been omitted, i.e., we omitted terms in the equations on the order of 0.001, then in the derived equations we should omit terms on the order 0.001. This means that the products of unknowns multiplied by non-dipole coefficients of approximation of intensity projections of the geomagnetic field onto the axes of an orbital system of coordinates must be omitted.

We will find the following equations:

$$
\left.
\begin{aligned}
D_{11}H^0_{21}-\frac{1}{2}D_{32}H^0_{32}&=H_{11}-H^0_{11},\\
D_{12}H^0_{21}-D_{31}H^0_{32}+\frac{1}{2}D_{35}H^0_{32}&=H_{12}-H^0_{12},\\
D_{13}H^0_{21}-\frac{1}{2}D_{34}H^0_{32}&=H_{13}-H^0_{13},\\
D_{14}H^0_{21}-\frac{1}{2}D_{33}H^0_{32}&=H_{14}-H^0_{14},\\
D_{15}H^0_{21}+\frac{1}{2}D_{32}H^0_{32}&=H_{15}-H^0_{15},\\
-\frac{1}{2}D_{35}H^0_{32}&=H_{16}-H^0_{16},\\
\frac{1}{2}D_{34}H^0_{32}&=H_{17}-H^0_{17};
\end{aligned}
\right\}
\qquad (11)
$$

$$
\left.
\begin{aligned}
D_{13}H^0_{13}-D_{22}H^0_{32}&=-2(H_{21}-H^0_{21}),\\
D_{14}H^0_{13}-2D_{21}H^0_{32}+D_{25}H^0_{32}&=-2(H_{22}-H^0_{22}),\\
2D_{11}H^0_{13}+D_{15}H^0_{13}-D_{24}H^0_{32}&=-2(H_{23}-H^0_{23}),\\
D_{12}H^0_{13}-D_{23}H^0_{32}&=-2(H_{24}-H^0_{24}),\\
D_{13}H^0_{13}+D_{22}H^0_{32}&=-2(H_{25}-H^0_{25}),\\
D_{14}H^0_{13}-D_{25}H^0_{32}&=-2(H_{26}-H^0_{26}),\\
D_{15}H^0_{13}+D_{24}H^0_{32}&=-2(H_{27}-H^0_{27});
\end{aligned}
\right\}
\qquad (12)
$$

$$\left.\begin{aligned}
-D_{21}H_{21}^{0}+\frac{1}{2}D_{33}H_{13}^{0}&=H_{31}-H_{31}^{0},\\
-D_{22}H_{21}^{0}+\frac{1}{2}D_{34}H_{13}^{0}&=H_{32}-H_{32}^{0},\\
-D_{23}H_{21}^{0}+D_{31}H_{13}^{0}+\frac{1}{2}D_{35}H_{13}^{0}&=H_{33}-H_{33}^{0},\\
-D_{24}H_{21}^{0}+\frac{1}{2}D_{32}H_{13}^{0}&=H_{34}-H_{34}^{0},\\
-D_{25}H_{21}^{0}+\frac{1}{2}D_{33}H_{13}^{0}&=H_{35}-H_{35}^{0},\\
\frac{1}{2}D_{34}H_{13}^{0}&=H_{36}-H_{36}^{0},\\
\frac{1}{2}D_{35}H_{13}^{0}&=H_{37}-H_{37}^{0}.
\end{aligned}\right| \tag{13}$$

System (11)-(13) contains 21 equations, the other 12 equa- /144 tions had to be omitted, since all therms of these equations have an order of 0.001. Thus, we have 21 equations to define 15 unknowns. But to within 12% accuracy the coefficients H_{13}^{0}, H_{21}^{0} and H_{32}^{0} on the left sides of equations (11)-(13) can be replaced by the dipole values

$$\left. H_{13}^{0}=H_0\sin i,\quad H_{21}^{0}=H_0\cos i,\quad H_{32}^{0}=-2H_0\sin i. \right| \tag{14}$$

Thus the substitution of formula (14) on the left sides of equations (11)-(13) yields an error on the order of 0.001. The matrix of coefficients of the obtained equations will coincide with the matrix of coefficients of equations of case A in study [1].

Thus, we have come to the conclusion that to within 0.0001 system (11)-(13) has only 12 independent equations. Thus the problem of selecting the zero approximation by magnetometer readings and using a non-dipole model of the geomagnetic field is not fully mathematically correct in the sense that the main terms of the equations do not define the angles uniquely. Of course, the adjunction of omitted low-precision equations makes the problem determined or even over-determined. But the use of a large number of low-precision equations can produce an effect not on the level of selecting a zero approximation, but in implementing the refining process by the method of least squares or some other statistical method.

We will transform the independent equations of system (11)-(13). The order of transformation is indicated by the right sides of the equations derived below:

$$\left.\begin{aligned}
D_{12}&=D_{31}\frac{H^0_{32}}{H^0_{21}}-\frac{1}{2}D_{35}\frac{H^0_{32}}{H^0_{21}}+\frac{H_{12}-H^0_{12}}{H^0_{21}},\\
D_{13}&=-\frac{H_{21}-H^0_{21}+H_{25}-H^0_{25}}{H^0_{13}},\\
D_{14}&=D_{21}\frac{H^0_{32}}{H^0_{13}}+\frac{H_{14}-H^0_{14}}{H^0_{21}}+\frac{H^0_{32}(H_{31}-H^0_{31})}{H^0_{13}H^0_{21}},\\
D_{15}&=-D_{11}+\frac{H_{11}-H^0_{11}+H_{15}-H^0_{15}}{H^0_{21}};
\end{aligned}\right| \tag{15}$$

$$\left.\begin{aligned}
D_{22}&=\frac{H_{21}-H^0_{21}-H_{25}+H^0_{25}}{H^0_{32}},\\
D_{23}&=D_{31}\frac{H^0_{13}}{H^0_{21}}+\frac{1}{2}D_{35}\frac{H^0_{13}}{H^0_{21}}-\frac{H_{33}-H^0_{33}}{H^0_{21}},\\
D_{24}&=D_{11}\frac{H^0_{13}}{H^0_{32}}-\frac{H^0_{13}(H_{11}-H^0_{11})}{H^0_{21}H^0_{32}}-\frac{H_{34}-H^0_{34}}{H^0_{21}},\\
D_{25}&=D_{21}+\frac{H_{31}-H^0_{31}-H_{35}+H^0_{35}}{H^0_{21}};
\end{aligned}\right| \tag{16}$$

$$\left.\begin{aligned}
&D_{32}=2D_{11}\frac{H^0_{21}}{H^0_{32}}-\frac{2(H_{11}-H^0_{11})}{H^0_{32}},\\
&D_{33}=2D_{21}\frac{H^0_{21}}{H^0_{13}}+\frac{2(H_{31}-H^0_{31})}{H^0_{13}},\\
&D_{34}=2\frac{H_{17}-H^0_{17}}{H^0_{32}},\quad D_{34}=2\frac{H_{36}-H^0_{36}}{H^0_{13}},\\
&D_{35}=-2\frac{H_{16}-H^0_{16}}{H^0_{32}},\quad D_{25}=2\frac{H_{37}-H^0_{37}}{H^0_{13}}.
\end{aligned}\right| \tag{17}$$

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Note 1. Other versions of transformed formulas are possible. In selecting transformations in the form (15)-(17) the goal was pursued of obtaining the smallest errors in computing the right sides. Thus with the aid of relations (11)-(13) the clear formula

$$\boxed{D_{35}=\frac{H_{12}-H^0_{12}}{H^0_{32}}+\frac{2H^0_{21}(H_{24}-H^0_{24})}{H^0_{13}H^0_{32}}+\frac{H_{33}-H^0_{33}}{H^0_{13}}.} \tag{18}$$

can be derived. On the basis of formula (18), the first formula of system (15) and the second formula of system (16) acquire a clearer appearance

$$D_{12}=D_{31}\frac{H_{32}^{0}}{H_{21}^{0}}+\frac{H_{12}-H_{12}^{0}}{2H_{21}^{0}}-\frac{H_{24}-H_{24}^{0}}{H_{13}^{0}}-\frac{H_{32}^{0}(H_{33}-H_{33}^{0})}{2H_{13}^{0}H_{21}^{0}}, \tag{19}$$

$$D_{23}=D_{31}\frac{H_{13}^{0}}{H_{21}^{0}}+\frac{H_{13}^{0}(H_{12}-H_{12}^{0})}{2H_{21}^{0}H_{32}^{0}}+\frac{H_{24}-H_{24}^{0}}{H_{32}^{0}}-\frac{H_{33}-H_{33}^{0}}{2H_{21}^{0}}. \tag{20}$$

But test computations showed that formula (18) can yield values which greatly differ from the values of the last formulas of notation (17).

Note 2. Let us consider the individual particular cases of satellite rotary motion. Given that the satellite makes tilting movements in the plane of the orbit. Then $\psi = 0$, $\phi = 0$ and consequently, $D_{1q} = 0$, $D_{2q} = 0$ ($q = 1, 2, 3, 4, 5$). Formulas (15)-(16) yield

$$D_{31}=\frac{1}{2}D_{35},\quad D_{32}=-2\frac{H_{11}-H_{11}^{0}}{H_{32}^{0}},\quad D_{33}=2\frac{H_{31}-H_{31}^{0}}{H_{13}^{0}}. \tag{21}$$

The last four formulas of notation (17) are also retained. Thus, the zero approximation of plane pitching oscillations is fully defined if it is true that $H_{13}^{0} \neq 0$ and $H_{32}^{0} \neq 0$. This means that the case $i = 0$ is excluded, i.e., the case of an equatorial orbit.

Given that $\theta = 0$, which means that $D_{3q} = 0$. By formulas (11) - (13) we will find that

$$\begin{aligned} D_{11}&=\frac{H_{11}-H_{11}^{0}}{H_{21}^{0}}, & D_{12}&=\frac{H_{12}-H_{12}^{0}}{H_{21}^{0}},\\ D_{13}&=\frac{H_{13}-H_{13}^{0}}{H_{21}^{0}}, & D_{14}&=\frac{H_{14}-H_{14}^{0}}{H_{21}^{0}},\\ D_{15}&=\frac{H_{15}-H_{15}^{0}}{H_{21}^{0}}, & D_{21}&=-\frac{H_{31}-H_{31}^{0}}{H_{21}^{0}},\\ D_{22}&=-\frac{H_{32}-H_{32}^{0}}{H_{21}^{0}}, & D_{23}&=-\frac{H_{33}-H_{33}^{0}}{H_{21}^{0}},\\ D_{24}&=-\frac{H_{34}-H_{34}^{0}}{H_{21}^{0}}, & D_{25}&=-\frac{H_{35}-H_{35}^{0}}{H_{21}^{0}}. \end{aligned} \tag{22}$$

Thus in this particular case, the zero approximation of angles ψ and ϕ is fully defined, if $H^0_{21} \neq 0$. Thereby, the case of a polar orbit i = 90° is excluded. /146

Note 3. From formulas (11)-(13) two quantities should vanish:

$$\xi = H^0_{13}(H_{14} - H^0_{14}) + 2H^0_{21}(H_{26} - H^0_{26}) + H^0_{32}(H_{35} - H^0_{35}) = 0, \tag{23}$$

$$\eta = H^0_{13}(H_{15} - H^0_{15}) + 2H^0_{21}(H_{27} - H^0_{27}) - H^0_{32}(H_{34} - H^0_{34}) = 0. \tag{24}$$

The fulfillment of formulas (23) and (24) by coefficients of the trigonometric approximation can serve as a check for applying the suggested formulas of zero approximation.

3. Optimization of the Solution of an Incompletely Correct Problem of Selecting the Zero Approximation

As follows from the discussion of the preceeding section, the correct definition of the coefficients of a trigonometric approximation of angles ϕ, ψ and θ requires additional knowledge of three quantities D_{11}, D_{21} and D_{31}. These quantities are averaged values of the angles. In several cases, the average values of the angles can be defined by averaged perturbing moments as a constant particular solution of linearized equations of motion. The equations of motion in this case will be equations of relative equilibrium of the satellite affected by gravitational, perturbing and other moments.

We can recommend a universal method of defining the coefficients D_{11}, D_{21}, and D_{31}. Since the obtained zero approximation will then be refined by statistical methods using all available information, it is desirable to have a zero approximation which is as close as possible to actual motion. We must bear in mind that coarse computations by formulas of the zero approximation will yield values which are chiefly greater than the actual oscillatory movements which the satellite experiences during oriented motion. Consequently, it will be wise to select such values of D_{11}, D_{21} and D_{31} as a zero approximation which yield for the use of formulas (15)-(17) the minimal value of the function

$$V = \sum_{p=1}^{3} \sum_{q=1}^{5} D_{pq}^2. \tag{25}$$

The partial derivative of V with respect to D_{11} appears as /147

$$\begin{aligned} \frac{\partial V}{\partial D_{11}} = 2\,(H_{32}^0)^{-2}\,\{D_{11}(H_{13}^{0^2}+4H_{21}^{0^2}+2H_{32}^{0^2}) - \\ -(H_{21}^0)^{-1}\,[(H_{13}^{0^2}+4H_{21}^{0^2}+H_{32}^{0^2})\,(H_{11}-H_{11}^0)+ \\ +H_{32}^{0^2}\,(H_{15}-H_{15}^0)+H_{13}^0H_{32}^0\,(H_{34}-H_{34}^0)]\}. \end{aligned} \tag{26}$$

We will derive an expression for the partial derivative of V with respect to D_{21}:

$$\begin{aligned} \frac{\partial V}{\partial D_{21}} = 2\,(H_{13}^0)^{-2}\,\{D_{21}\,(2H_{13}^{0^2}+4H_{21}^{0^2}+H_{32}^{0^2}) + \\ +(H_{21}^0)^{-1}\,H_{13}^0H_{32}^0\,(H_{14}+H_{14}^0)+(H_{13}^{0^2}+4H_{21}^{0^2}+H_{32}^{0^2})\,(H_{31}-H_{31}^0)- \\ -H_{13}^{0^2}\,(H_{35}-H_{35}^0)]\}. \end{aligned} \tag{27}$$

Let us find an expression for the partial derivative of V with respect to D_{31}:

$$\begin{aligned} \frac{\partial V}{\partial D_{31}} = 2\,(H_{21}^0)^{-1}\,[D_{31}\,(H_{13}^{0^2}+H_{21}^{0^2}+H_{32}^{0^2}) + \\ +H_{32}^0\,(H_{12}-H_{12}^0) + \frac{1}{2}D_{35}\,(H_{13}^{0^2}-H_{32}^{0^2})-H_{13}^0\,(H_{33}-H_{33}^0)]. \end{aligned} \tag{28}$$

Formulas (26)-(28) show that mixed derivatives are equal to zero, and all double derivatives with respect to the same unknown are positive. Thus the roots of the right sides of relations

(26)-(28) yield the minimum. In view of the linearity of the right sides of these relations, this minimum is the only one.

Let us equate to zero the right sides of relations (26)-(28)

$$D_{11}=\frac{(H_{13}^{0^2}+4H_{21}^{0^2}+H_{32}^{0^2})(H_{11}-H_{11}^{0})+H_{32}^{0^2}(H_{15}-H_{15}^{0})+H_{13}^{0}H_{32}^{0}(H_{34}-H_{34}^{0})}{H_{21}^{0}(H_{13}^{0^2}+4H_{21}^{0^2}+2H_{32}^{0^2})}, \tag{29}$$

$$D_{21}=\frac{-H_{13}^{0}H_{32}^{0}(H_{14}-H_{14}^{0})-(H_{13}^{0^2}+4H_{21}^{0^2}+H_{32}^{0^2})(H_{31}-H_{31}^{0})+H_{13}^{0^2}(H_{35}-H_{35}^{0})}{H_{21}^{0}(2H_{13}^{0^2}+4H_{21}^{0^2}+H_{32}^{0^2})}. \tag{30}$$

$$D_{31}=\frac{-H_{32}^{0}(H_{12}-H_{12}^{0})+\frac{1}{2}D_{35}(H_{32}^{0^2}-H_{13}^{0^2})+H_{13}^{0}(H_{33}-H_{33}^{0})}{H_{13}^{0^2}+H_{21}^{0^2}+H_{32}^{0^2}}, \tag{31}$$

Formulas (29) and (30) lose their meaning when $H_{21}^{0}=0$, i.e., in a polar orbit.

4. Solving the Problem of Selecting a Zero Approximation with Additional Information

1. Let us assume that, as in study [1], we know the values of angles ψ, ϕ and θ for several points. Let us employ formulas /148 of that study. Instead of equations (44)-(46) of study [1], we find that

$$\begin{aligned} &D_{11}\sin u_k+D_{14}\cos u_k+\frac{1}{2}D_{23}H_{32}^{0}(H_{13}^{0})^{-1}= \\ &=\frac{1}{2\sin u_k}\Big[\psi(u_k)-D_{13}\cos u_k-\frac{\cos 2u_k}{H_{21}^{0}}(H_{11}-H_{11}^{0}+H_{15}-H_{15}^{0})+ \\ &+2\frac{\sin u_k}{H_{13}^{0}}(H_{24}-H_{24}^{0})\Big]=\Psi_k, \end{aligned} \tag{32}$$

$$\begin{aligned} &D_{11}\sin u_k+D_{14}\cos u_k+\frac{1}{2}D_{23}H_{32}^{0}(H_{13}^{0})^{-1}= \\ &=\frac{H_{32}^{0}}{2H_{13}^{0}\cos u_k}\Big[\varphi(u_k)-D_{22}\sin u_k+\frac{\cos 2u_k}{H_{21}^{0}}(H_{35}-H_{35}-H_{31}+H_{31}^{0})- \\ &-\frac{H_{13}^{0}(H_{14}-H_{14}^{0})+H_{32}^{0}(H_{31}-H_{31}^{0})}{H_{21}^{0}H_{32}^{0}}(1+\cos 2u_k)+ \\ &+\frac{H_{13}^{0}(H_{11}-H_{11}^{0})+H_{32}^{0}(H_{34}-H_{34}^{0})}{H_{21}^{0}H_{32}^{0}}\sin 2u_k\Big]=\Phi_k, \end{aligned} \tag{33}$$

$$D_{11}\sin u_k+D_{14}\cos u_k+\frac{1}{2}D_{23}H_{32}^0(H_{13}^0)^{-1}=$$
$$=\frac{H_{32}^0}{2H_{21}^0}\Big[\theta(u_k)-D_{34}\sin 2u_k-D_{35}\left(\frac{1}{2}+\cos 2u_k\right)+$$
$$+\frac{2\sin u_k}{H_{32}^0}(H_{11}-H_{11}^0)+\frac{H_{12}-H_{12}^0}{H_{32}^0}+$$
$$+\frac{2\cos u_k}{H_{32}^0}(H_{14}-H_{14}^0)+\frac{2H_{21}^0}{H_{13}^0H_{32}^0}(H_{24}-H_{24}^0)\Big]=\Theta_k(u_k). \qquad (34)$$

On the average, system (32)-(34) is equivalent to a single equation:

$$D_{11}\sin u_k+D_{14}\cos u_k+\frac{1}{2}D_{23}H_{32}^0(H_{13}^0)^{-1}=Q_k, \qquad (35)$$

where, as in formula (47) of study [1],

$$Q_k=\frac{1}{3}(\Psi_k+\Phi_k+\Theta_k). \qquad (36)$$

The coefficients D_{11} and D_{14} will be defined by formulas (50) and (51) of the cited work, while coefficient D_{23} in terms of the type of formula (52) of this study will appear as

$$D_{23}=-\frac{2H_{13}^0}{3H_{32}^0}[D_{11}(\sin u_1+\sin u_2+\sin u_3)+$$
$$+D_{14}(\cos u_1+\cos u_2+\cos u_3)-Q_1-Q_2-Q_3]. \qquad (37)$$

Taking into account formulas (15-17) and the formulas just mentioned, we find that

$$D_{11}=\frac{Q_1(\cos u_3-\cos u_2)+Q_2(\cos u_1-\cos u_3)+Q_3(\cos u_3-\cos u_1)}{\sin(u_3-u_2)+\sin(u_1-u_3)+\sin(u_2-u_1)}, \qquad (38)$$
$$D_{21}=-\frac{H_{13}^0}{H_{21}^0H_{32}^0}(H_{14}-H_{14}^0)-\frac{H_{31}-H_{31}^0}{H_{21}^0}-\Big/\frac{H_{13}^0}{H_{32}^0}.$$

$$\frac{Q_1(\sin u_3-\sin u_2)+Q_2(\sin u_1-\sin u_3)+Q_3(\sin u_2-\sin u_1)}{\sin(u_3-u_2)+\sin(u_1-u_3)+\sin(u_2-u_1)}, \qquad (39)$$

$$D_{31}=-\frac{1}{2}D_{35}+\frac{H_{21}^0}{H_{13}^0}D_{23}+\frac{H_{33}-H_{33}^0}{H_{13}^0}. \qquad (40)$$

In the notion of (40), the coefficient D_{23} must be expressed through formula (37). /149

Thus, if we know the values of angles at three points in the orbit which have mutual angular distances not equal to 180°, the zero approximation, as shown by formulas (38)-(40), is uniquely defined with the exception of cases i = 0° and i = 90°.

2. Given that we now know the direction of the second vector at several points in the orbit. Instead of equation (67) of study [1], we will find that

$$\begin{aligned}
&D_{11}\sin u_k + D_{14}\cos u_k + \frac{1}{2}D_{23}H^0_{32}(H^0_{13})^{-1} = \\
&= \frac{1}{2}[\sin u_k \cos(S_k, y_0) - H^0_{21}(H^0_{32})^{-1}\cos(S_k, z_0)]^{-1}\times \\
&\times\{\cos(S_k, x) - \cos(s_k, x_0) - D_{13}\cos u_k \cos(S_k, y_0) + \\
&+ D_{34}\sin 2u_k \cos(S_k, z_0) + D_{35}\left[\frac{H^0_{32}}{H^0_{21}}\cos(S_k, y_0) + \right. \\
&\left. + \left(-\frac{1}{2} + \cos 2u_k\right)\cos(S_k, z)\right] - \\
&- \cos(S_k, y_0)\left[\frac{H^0_{32}}{H^0_{21}}\sin u_k\left(\frac{H_{12}-H^0_{12}}{H^0_{32}} + \frac{H_{33}-H^0_{33}}{H^0_{13}}\right) + \right. \\
&\left. + \cos 2u_k \frac{H_{11}-H^0_{11}+H_{15}-H^0_{15}}{H^0_{21}}\right] + \cos(S_k, z_0)\left(\frac{H_{33}-H^0_{33}}{H^0_{13}} - \right. \\
&\left. - 2\sin u_k \frac{H_{11}-H^0_{11}}{H^0_{21}} - 2\cos u_k \frac{H_{14}-H^0_{14}}{H^0_{32}}\right)\} = \Psi'_k.
\end{aligned} \qquad (41)$$

Further, instead of equation (68) of the cited work, we will find that

$$\begin{aligned}
&D_{11}\sin u_k + D_{14}\cos u_k + \frac{1}{2}D_{23}H^0_{32}(H^0_{13})^{-1} = \\
&= -\frac{1}{2}[\sin u_k \cos(S_k, x_0) - H^0_{13}(H^0_{32})^{-1}\cos u_k \cos(S_k, z_0)]^{-1}\times \\
&\times\{\cos(S_k, y) - \cos(s_k, y_0) + D_{13}\cos u_k \cos(S_k, x_0) - \\
&- D_{22}\sin u_k \cos(S_k, z_0) - D_{35}\frac{H^0_{32}}{H^0_{21}}\sin u_k \cos(S_k, x_0) + \\
&+ \cos(S_k, z_0)\left\{\frac{H_{31}-H^0_{31}}{H^0_{21}} + 2\frac{H^0_{13}(H_{14}-H^0_{14})}{H^0_{21}H^0_{32}}\cos^2 u_k + \right. \\
&\left. + \frac{H^0_{13}(H_{11}-H^0_{11}) + H^0_{32}(H_{34}-H^0_{34})}{H^0_{21}H^0_{32}}\sin 2u_k + \frac{H_{35}-H^0_{35}}{H^0_{21}}\cos 2u_k\right\} + \\
&+ \cos(S_k, x_0)\left[\frac{H^0_{13}(H_{12}-H^0_{12}) + H^0_{32}(H_{33}-H^0_{33})}{H^0_{13}H^0_{21}}\sin u_k + \right. \\
&\left. + \frac{H_{11}-H^0_{11}+H_{15}-H^0_{15}}{H^0_{21}}\cos 2u_k\right]\} = \Phi'_k.
\end{aligned} \qquad (42)$$

Finally, instead of equation (69) of study [1] we will find that

$$
\begin{aligned}
& D_{11} \sin u_k + D_{14} \cos u_k + \frac{1}{2} D_{23} H_{32}^0 (H_{13}^0)^{-1} = \\
& = -\frac{1}{2} \frac{H_{32}^0}{H_{13}^0} \left[\cos u_k \cos (S_k, y_0) - \frac{H_{21}^0}{H_{13}^0} \cos (S_k, x_0) \right]^{-1} \times \\
& \times \Big\{ \cos (S_k, z) - \cos (S_k, z_0) + D_{22} \sin u_k \cos (S_k, y_0) - \\
& - D_{34} \sin 2u_k \cos (S_k, x_0) + D_{35} \left(\frac{1}{2} - \cos 2u_k \right) \cos (S_k, x_0) + \\
& + \cos (S_k, x_0) \left(-\frac{H_{33} - H_{33}^0}{H_{13}^0} + 2 \frac{H_{11} - H_{11}^0}{H_{32}^0} \sin u_k + \right. \\
& \left. + 2 \frac{H_{14} - H_{14}^0}{H_{32}^0} \cos u_k \right) - \cos (S_k, y_0) \left[\frac{H_{31} - H_{31}^0}{H_{21}^0} + \right. \\
& + 2 \frac{H_{13}^0 (H_{14} - H_{14}^0)}{H_{32}^0 H_{21}^0} \cos^2 u_k + \frac{H_{13}^0 (H_{11} - H_{11}^0) + H_{32}^0 (H_{34} - H_{31}^0)}{H_{21}^0 H_{32}^0} \sin 2u_k + \\
& \left. + \frac{H_{35} - H_{35}^0}{H_{21}^0} \cos 2u_k \right] \Big\} = \Theta_k'.
\end{aligned}
\tag{43}
$$

We know assume the notation

$$
\boxed{Q_k' = \frac{1}{3} (\Psi_k' + \Phi_k' + \Theta_k').}
\tag{44}
$$

Unknown coefficients D_{11}, D_{21} and D_{31} will be defined by formulas (38)-(40). In this context, Q_k is replaced by Q_k'.

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